Effect of black hole charge on acceleration of particles in magnetized Kerr spacetime

Ondřej Kopáček[1], Vladimír Karas[2]

[1]Faculty of Science, Humanities and Education, Technical University of Liberec, Czech Republic

[2]Astronomical Institute, Czech Academy of Sciences, Prague, Czech Republic

An interplay of magnetic fields and gravitation drives accretion and outflows near black holes. In particular, for the acceleration of electrically charged particles and their collimation in jets, an ordered component of the magnetic field seems to be essential. Here, we discuss the role of the combined effects of black hole charge and large-scale, generally non-axisymmetric, magnetic fields in transporting the charged particles and dust grains from the bound orbits in the equatorial plane of a rotating (Kerr) black hole and the resulting acceleration along jet-like trajectories escaping the system. We consider a specific scenario of destabilization of circular geodesics of initially neutral matter by charging (e.g., due to photoionization). Some particles may be set on escaping trajectories and attain relativistic velocity. We investigate the system numerically and construct the escape-boundary plots, which show the location and the extent of the escape zones. We focus on the effect of black hole charge on the formation and location of escape zones, and compute the maximal escape velocity.

Introduction

This work represents the continuation of our steady effort [1–4] to reveal dynamic properties of electrically charged matter being exposed to the simultaneous action of strong gravitational and electromagnetic fields surrounding compact objects – neutron stars and black holes. Survey of the test particle trajectories might be regarded as the one particle approximation to the complex dynamics of the astrophysical plasma, which is applicable once the plasma is sufficiently diluted. Regions of diluted plasma are likely to be found above and below the main accretion body of astrophysical systems driven by compact objects.

We numerically investigate the dynamics of charged test particles in the vicinity of a rotating electrically charged black hole embedded in the external large-scale magnetic field inclined with respect to the rotation axis. In particular, we discuss under which circumstances the electrically neutral material initially bound to stable orbits in the equatorial plane may be destabilized by the charging process and set on the escaping trajectories, liberated from the attraction of the center, and even accelerated to relativistic velocities.

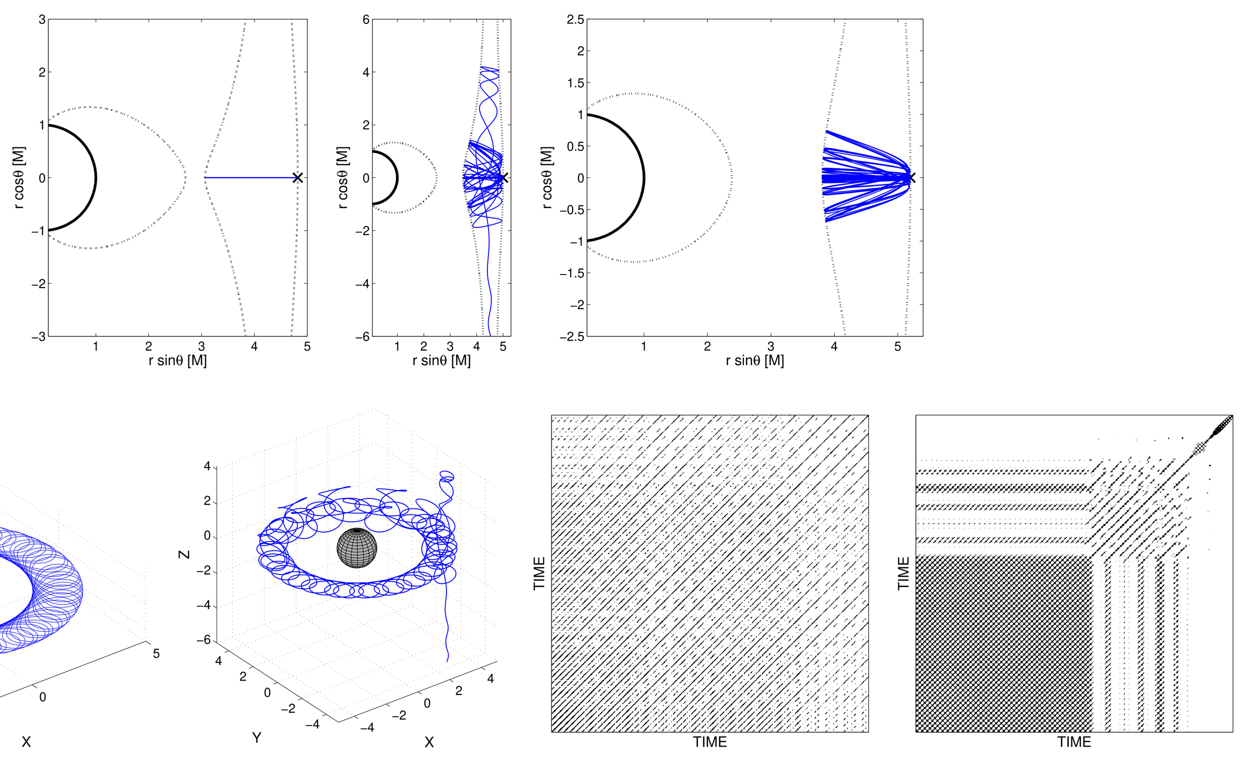

Fig. 1: Typical trajectories encountered in the escape zone. The upper left panel ($r_0 = 4.82$) shows the stable orbit oscillating in the equatorial plane, the upper middle panel illustrates the escaping particle charged at $r_0 = 5$, while the upper right panel shows the stable orbit oscillating around the equatorial plane ($r_0 = 5.2$). Parameters of the system are $a = 1$, $qB_z = -1$ ($B_x = 0$), and $qQ = 0$. The initial radius of the Keplerian orbit is marked by the cross. The solid black line denotes the horizon of the black hole. Dashed lines show relevant isopotential contours of the effective potential. The allowed region for escaping trajectories is bound between the two rotational iso-surfaces forming the open corridor along the symmetry axis ($z \equiv r\cos\theta$). In the equatorial plane, the inner iso-surface bends toward the horizon, but they both become perfectly cylindrical and parallel farther from the black hole. The first two trajectories from the upper panels are shown in a 3D view in the bottom panels. Their recurrence plots [5] are also presented, showing the regular dynamics of the stable orbit (the third bottom panel) and chaotic nature of the escaping orbit (the fourth bottom panel).

Weakly charged rotating black hole in external nonaxisymmetric magnetic field

Kerr metric describing the geometry of the spacetime around the rotating black hole may be expressed in Boyer-Lindquist coordinates $x^\mu = (t,\ r,\ \theta,\ \varphi)$ as follows [6]:

$$ds^2 = -\frac{\Delta}{\Sigma}\left[dt - a\sin\theta\, d\varphi\right]^2 + \frac{\sin^2\theta}{\Sigma}\left[(r^2+a^2)d\varphi - a\, dt\right]^2 + \frac{\Sigma}{\Delta}\, dr^2 + \Sigma d\theta^2,$$

where

$$\Delta \equiv r^2 - 2Mr + a^2, \quad \Sigma \equiv r^2 + a^2\cos^2\theta.$$

Geometrized units are used throughout; values of basic constants equal unity $G = c = k = k_C = 1$.

We employ the test-field solution of Maxwell's equations for a weakly magnetized and weakly charged Kerr black hole immersed in an asymptotically uniform magnetic field specified by the component B_z parallel to the spin axis and the perpendicular component B_x. The electromagnetic vector potential A_μ is given as follows [7]:

$$\begin{aligned}
A_t &= \frac{B_z a M r}{\Sigma}\left(1+\cos^2\theta\right) - B_z a + \frac{B_x a M \sin\theta\cos\theta}{\Sigma}\left(r\cos\psi - a\sin\psi\right) - \frac{Qr}{\Sigma},\\
A_r &= -B_x(r-M)\cos\theta\sin\theta\sin\psi,\\
A_\theta &= -B_x a(r\sin^2\theta + M\cos^2\theta)\cos\psi - B_x(r^2\cos^2\theta - Mr\cos 2\theta + a^2\cos 2\theta)\sin\psi,\\
A_\varphi &= B_z\sin^2\theta\left[\frac{1}{2}(r^2+a^2) - \frac{a^2 M r}{\Sigma}(1+\cos^2\theta)\right] - B_x\sin\theta\cos\theta\left[\Delta\cos\psi + \frac{(r^2+a^2)M}{\Sigma}\left(r\cos\psi - a\sin\psi\right)\right] + \frac{Qar\sin^2\theta}{\Sigma},
\end{aligned}$$

where ψ denotes the azimuthal coordinate of Kerr ingoing coordinates and Q corresponds to the charge of the Kerr black hole. The charge is supposed to be small enough $Q \ll M$ so that its effect on geometry may be neglected (i.e., we do not employ the metric of Kerr-Newman solution, although the relevant terms of the vector potential are identical to this solution). In general, the resulting electromagnetic field is described by three independent parameters B_x, B_z, and Q. Alternatively, the asymptotic magnetic field may also be parametrized by the field strength $B = \sqrt{B_x^2 + B_z^2}$ and inclination angle $\alpha \equiv \arctan\left(B_x/B_z\right)$.

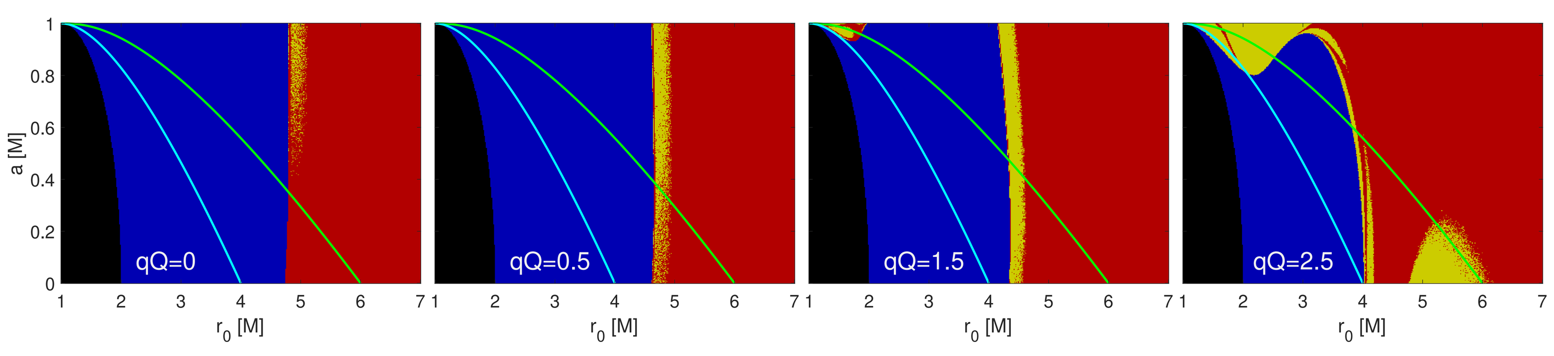

Fig. 2: Evolution of the escape zones of charged particles affected by the Coulomb repulsion of a charged black hole in an aligned magnetic field ($B_x = 0$). Color-coding of trajectories: blue for plunging orbits, red for stable ones (bound to the black hole), and yellow for escaping trajectories. The inner black region marks the horizon of the black hole. The green line denotes the ISCO of neutral particles, while the cyan line marks a marginally bound circular orbit. Parameters of the system are $qB = -1$ and inclination $\alpha = 0$ while qQ varies as indicated.

Conclusions

We have analyzed the effect of the black hole charge on the outflow of matter from the inner part of the magnetized accretion disk, driven by the charging of initially neutral accreted dust. Previously, we have shown that a rather simple setup is sufficient to model an outflow even in the single particle approximation, where magnetohydrodynamic terms are neglected. Indeed, it appeared that essential and sufficient ingredients to allow the matter to escape are the rotation of the central black hole, $a \neq 0$, and the presence of a uniform magnetic field B. We assumed a basic accretion scenario [8] with the matter slowly sinking between circular Keplerian orbits (and free-falling below the ISCO) in the equatorial plane, until it reaches charging radius r_0, where it obtains a nonzero charge q. In general, the charge may stabilize the plunging orbit and possibly turn a stable orbit into a plunging one. However, the analysis of the effective potential reveals that, under certain conditions, the charged particles may also be accelerated to high energies and escape to infinity.

Here, we focus on escaping particles (typically forming escape zones near the horizon) and further generalize the model by considering a nonzero electric charge of the spinning black hole and an oblique magnetosphere with an arbitrary inclination of the asymptotically uniform magnetic field with respect to the spin axis. In particular, we study the effect of the black hole charge Q and show how it affects the formation of the escape zones and acceleration of escaping particles.

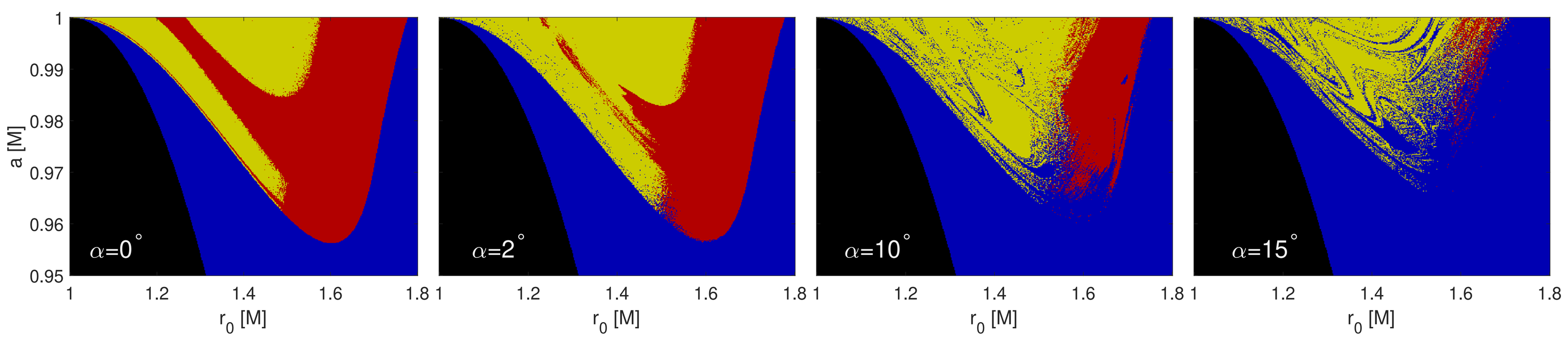

Fig. 3: The Innermost Escape Zone induced due to Coulomb interaction affected by the inclination of the magnetic field. Color-coding of trajectories is the same as in Fig. 2. Parameters of the system are $qB = -1$, $qQ = 1.2$, $\varphi(0) = 0$, and inclination angle α varies as indicated.

Equations of motion

Hamiltonian describing the motion of the ionized test particle of charge q is given as follows [6]:

$$\mathcal{H} = \frac{1}{2} g^{\mu\nu}(\pi_\mu - qA_\mu)(\pi_\nu - qA_\nu),$$

where π_μ is the generalized (canonical) momentum and A_μ denotes the vector potential related to the electromagnetic field tensor as $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$.

Hamiltonian equations of motion are given as:

$$dx^\mu/d\lambda = \partial\mathcal{H}/\partial\pi_\mu, \quad d\pi_\mu/d\lambda = -\partial\mathcal{H}/\partial x^\mu,$$

where $\lambda = \tau/m$ is the affine parameter, τ the proper time and m represents the rest mass of the particle.

The second Hamilton's equation ensures that the time component of the four-momentum $\pi_t = p_t + qA_t \equiv -E$ represents a constant of motion (energy E). In the special case of an axisymmetric magnetosphere ($B_x = 0$), the azimuthal component $\pi_\varphi = p_\varphi + qA_\varphi \equiv L$ is also conserved. However, even in this case, there are more degrees of freedom than constants of motion since the Carter constant is not conserved anymore due to the magnetic field. As a result, the system is not fully integrable and exhibits deterministic chaos, which becomes even stronger in the general non-axisymmetric system where the angular momentum L is also not conserved [3]. Numerical integration of Hamilton's equations is carried out using the multistep Adams-Bashforth-Moulton solver of variable order. In several cases when higher accuracy is demanded, we employ the 7th- 8th order Dormand-Prince method.

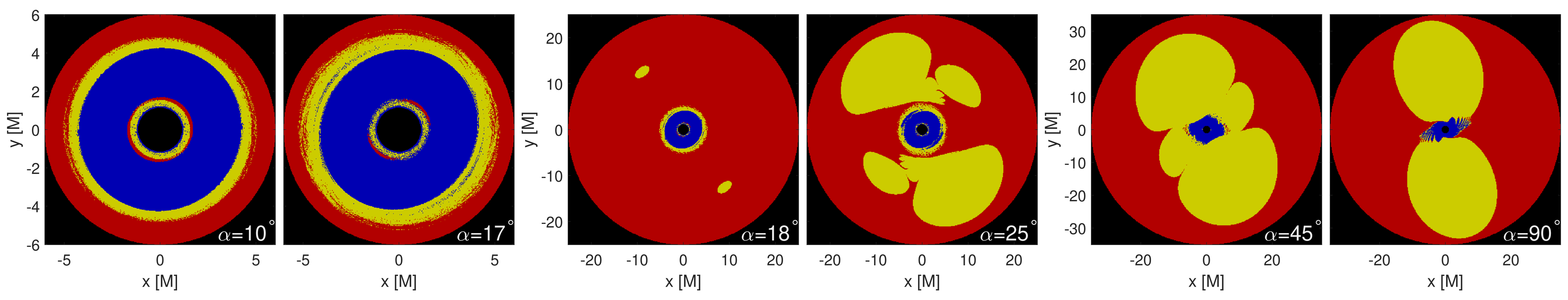

Fig. 4: Escape zones affected by the inclination of the magnetic field. Parameters of the system are $qB = -1$, $qQ = 1.2$, $a = 0.99$, and inclination angle α varies as indicated in each panel.

Effective potential

Introducing the particle charge q, the value of the effective potential changes accordingly. In order to study trajectories of escaping particles, we examine the behavior of the potential for $r \gg M$. In particular, for the initially neutral particle on circular (Keplerian) orbit [9] with energy E_{Kep} ionized in the equatorial plane at r_0, we obtain the following relation valid in the asymptotic region:

$$E - V_{\mathrm{eff}}|_{r\gg M} = E_{\mathrm{Kep}} - 1 + \frac{qQ - qB_z a}{r_0} + \mathcal{O}\left(r^{-1}\right).$$

As the motion is possible only for $E \geq V_{\mathrm{eff}}$, the right-hand side of the above equation needs to remain nonnegative in order to make the asymptotic region accessible. Since $E_{\mathrm{Kep}} < 1$ with finite r_0 and $a \geq 0$ is considered (without loss of generality), we observe that particles may only escape for $qQ > qB_z a$ and that the asymptotic velocity of escaping particles is a decreasing function of r_0 expressed by the final (asymptotic) value of Lorentz factor as follows:

$$\gamma^* = E_{\mathrm{Kep}} + \frac{qQ - qB_z a}{r_0}.$$

Unlike the neutral black hole studied in [1, 2], the escape of particles remains possible even for the non-spinning and/or non-magnetized black hole, provided that $qQ > 0$ (i.e., charges q and Q are of the same sign, resulting in Coulomb repulsion).

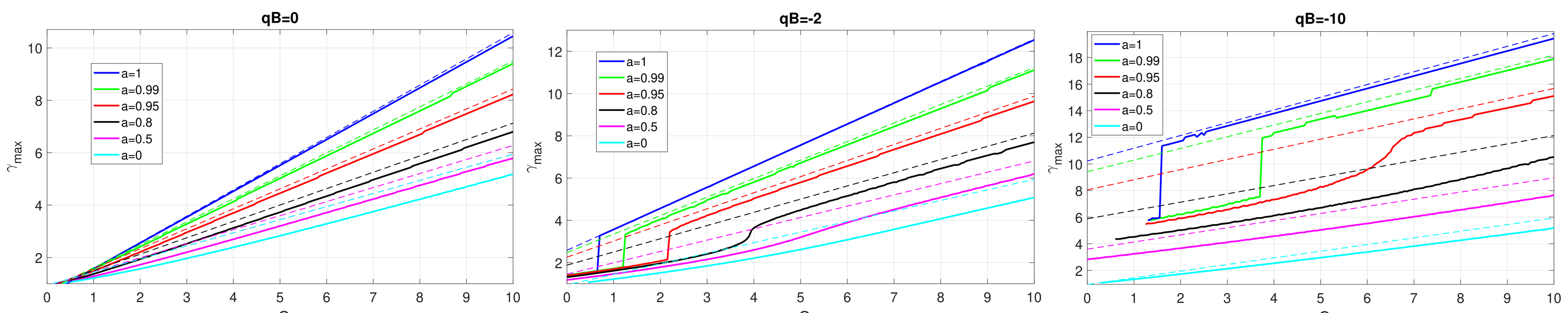

Fig. 5: Maximum Lorentz factor $\gamma_{\max}$ achieved within the escape zones as a function of charge product qQ compared with a theoretical limit γ^* determined from the asymptotics of the effective potential (dashed lines).

References

[1] Kopáček, O. and Karas, V.: Near-horizon Structure of Escape Zones of Electrically Charged Particles around Weakly Magnetized Rotating Black Hole. II. Acceleration and Escape in the Oblique Magnetosphere, *The Astrophysical Journal*, 900, 119, 2020

[2] Kopáček, O. and Karas, V.: Near-horizon escape zones of electrically charged particles around weakly magnetized rotating black hole, *The Astrophysical Journal*, 853, 53, 2018

[3] Kopáček, O. and Karas, V.: Inducing Chaos by Breaking Axial Symmetry in a Black Hole Magnetosphere, *The Astrophysical Journal*, 787, 117, 2014

[4] Kopáček, O., Karas, V., Kovář, J. and Stuchlík, Z.: Transition from Regular to Chaotic Circulation in Magnetized Coronae near Compact Objects, *The Astrophysical Journal*, 722, 1240-1259, 2010

[5] Marwan, N., Carmen Romano, M., Thiel, M. and Kurths, J.: Recurrence plots for the analysis of complex systems, *Physics Reports*, 438, 237-329, 2007

[6] Misner C. W., Thorne K. S. and Wheeler J.A.: Gravitation, Freeman, San Francisco, 1973

[7] Bičák, J. and Janiš, V.: Magnetic fluxes across black holes, *Monthly Notices of the Royal Astronomical Society*, 212, 899-915, 1985

[8] Novikov, I. D., & Thorne, K. S. 1973, in Black Holes - Les Astres Occlus, eds. C. De Witt and B. S. De Witt (New York: Gordon & Breach), pp. 343-450

[9] Bardeen, J. M., Press, W. H., & Teukolsky, S. A.: Rotating Black Holes: Locally Nonrotating Frames, Energy Extraction, and Scalar Synchrotron Radiation, *The Astrophysical Journal*, 178, 347-370, 1972

[10] Wald, R. M.: Black hole in a uniform magnetic field, *Physical Review D*, 6, 1680, 1974

E-mail: ondrej.kopacek@tul.cz