From Single Explosions to Macroscopic Viscosity

Angular Momentum Redistribution by SN Feedback

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Background

The growth of supermassive black holes (SMBHs) requires massive, sustained gas inflows. However, it remains unclear how mass is transported from the 10–100 pc scales of the Central Molecular Zone, through the Circumnuclear Disk, and onto the SMBH. While macroscopic transport can be parameterized, the actual physical drivers of the underlying turbulence in these extreme environments remain heavily debated.

Our Approach

We investigate supernova (SN) feedback as the primary driver of stochastic turbulence. We treat the average ambient SN rate per unit area ($\mathcal{R}_{\rm SN}$) as a continuous momentum injector. We analytically derive a macroscopic effective viscosity driven entirely, and self-consistently, by SN explosions.

Mixing Radius

To calculate a macroscopic viscosity, we must first define the characteristic length scale of individual explosions. In dense galactic center environments, SN thermal energy radiates away rapidly. The remnant expands predominantly via momentum conservation (the snowplow phase). The swept-up shell effectively stalls and mixes when its velocity drops to match the ambient 1D turbulent velocity dispersion (σ).

For a uniform medium, this stopping condition yields the mixing radius

$$R_{\rm mix} = \left(\frac{3 p_{\rm SN}}{4\pi\rho\sigma} \right)^{1/3},$$

where ρ is the ambient density and $p_{\rm SN}$ is the terminal injected momentum (e.g., Kim & Ostriker 2015).

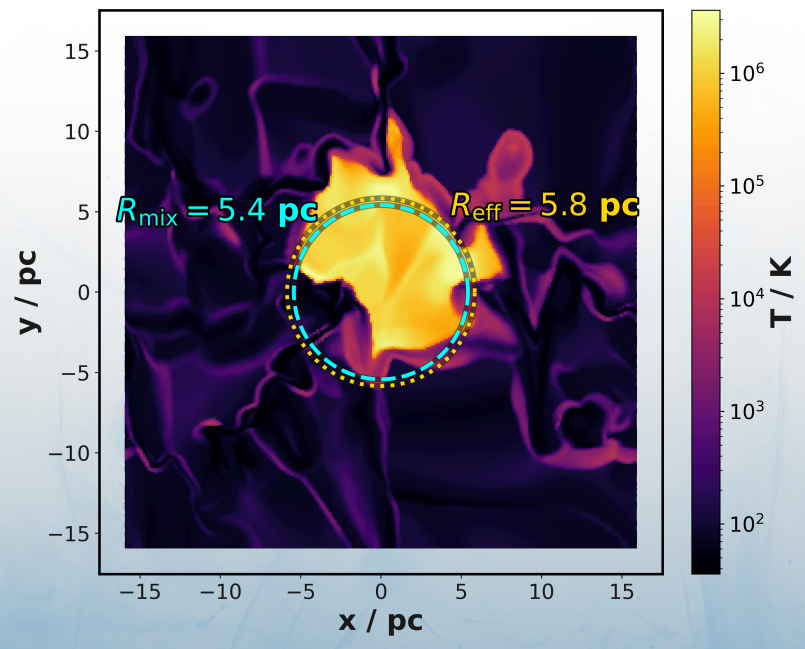


Figure: Our 3D simulation of an SN blast wave in a turbulent medium just before mixing. $R_{\rm eff}$ is defined as the equivalent spherical radius of the irregular 3D swept-up blast volume.

The Macroscopic Transport Model

We calculate the net angular momentum balance across a fixed radial boundary R. We get an integro-differential master equation for the conserved angular momentum density, $\Psi = \Sigma h$,

$$\frac{\partial \Psi}{\partial t} = \int_0^\infty [W(R_0|R)\Psi(R) - W(R|R_0)\Psi(R_0)]dR\,,$$

where $W(R_0|R)$ is the transition rate of material moving from R_0 to R, Σ is the surface density, and h is the specific angular momentum.

By performing a Taylor expansion on the master equation, we derive the Kramers–Moyal coefficients, $D^{(n)}$. The second-order diffusion term naturally manifests as a macroscopic effective viscosity, $\nu = D^{(2)}/2$,

$$\frac{\partial \Psi}{\partial t} \approx \frac{\partial^2}{\partial R^2}\left[\frac{D^{(2)}(R)}{2}\Psi(R)\right].$$

Projecting the 3D isotropic expansion onto the 1D radial transport axis isolates the purely radial mass flux. Depending on the disk geometry, this yields a self-consistent SN-driven viscosity

$$\nu_{\rm c} = \frac{1}{4} R_{\rm mix}^2 \mathcal{R}_{\rm SN} M_{\rm disp}\,, \quad \text{(Cylindrical Ejecta)}$$

$$\nu_{\rm s} = \frac{1}{6} R_{\rm mix}^2 \mathcal{R}_{\rm SN} M_{\rm disp}\,, \quad \text{(Spherical Ejecta)}$$

where $M_{\rm disp} = p_{\rm SN}/\sigma$ is the total mass of the swept-up shell. This yields an accretion rate of $\dot{M}_{\rm visc} = 2\pi\nu\Sigma$ for a steady-state disk with a flat rotation curve.

Breaking Symmetry: Advection Drift

While viscosity is always present, a non-zero first-order term, $D^{(1)}$, arises only when the symmetry of mass exchange is broken by a density gradient ($R_{\rm mix} \sim l_\rho$). This asymmetry generates a drift velocity that transports mass down the gradient, completely independent of standard viscous spreading

$$D_{\rm c}^{(1)} = -\frac{1}{6}\frac{\mathcal{R}_{\rm SN} M_{\rm disp} l_0^2}{l_\rho}\,.$$

This results in a direct advective accretion rate of $\dot{M}_{\rm adv} = 2\pi R\Sigma D^{(1)}$, accelerating inflows where density gradients are steep.

Conclusions

- Our turbulent simulations confirm that the 1D analytical stopping condition accurately predicts the maximum 3D volumetric extent of the blast wave ($R_{\rm eff}$) prior to mixing.
- Using the Kramers–Moyal expansion of the master equation, we derive a macroscopic effective viscosity ($\nu \propto R_{\rm mix}^2$) driven purely by stochastic SN explosions.
- In regions with steep density gradients, the symmetry of mass exchange is broken. This creates a drift velocity that transports mass down the density gradient, driving accretion independent of normal viscous spreading.

References:

Kim, C.-G., & Ostriker, E. C. (2015). ApJ, 802(2), 99.

Lynden-Bell, D., & Pringle, J. E. (1974). MNRAS, 168, 603-637.

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