Habilitation Thesis Reviewer's Report

Masaryk University	
Faculty	Faculty of Science
Procedure field	Algebra and number theory
Applicant	Bc. Lukáš Vokřínek, PhD
Applicant's home unit, institution	Department of Mathematics and Statistics, Faculty of Science MU
Habilitation thesis	Algorithmic aspects of topological problems
Reviewer	Shmuel Weinberger
Reviewer's home unit, institution	University of Chicago

Algebraic topology is a central area of modern mathematics, and largely deals with spaces of continuous functions from one space to another. It is a remarkable fact, that is the result of decades of work, that frequently such analyses are possible. (Despite this success, there is no non-contractible simply connected complex, a of whose homotopy groups are known.)

Besides its intrinsic interest many other areas feed from algebraic topology, with the goal of reducing problems to ones of algebraic topology and solving those. Applications can be found in geometric topology, algebraic geometry, combinatorics, theoretical computer science, analysis and beyond. These applications crucially depend on actually being able to solve algebraic topology probems.

The situation on the ground is this: There is an abstract theorem of Brown that asserts that simply connected homotopy theory is substantially algorithmic, but the agorithms (as analyzed by Gromov) are typically towers of exponentials. In practice, interesting problems do frequently get solved over the course of human lifetimes -- although using a mixture of the ideas that Brown depended on and others, together with much ingenuity.

Vokrinek's work is a cutting edge analysis of the complexity of algebraic topology and of embedding theory (which is one of the areas where algebraic topology has very strong impications). It contains specific algorithms (that suffice for metastable embedding theory), an important extension of the work of Brown (going from homotopy groups to maps of spaces), an improvement of his algorithm for computing Postnikov systems to polynomial time (which one can hope will usher in the day of quick practical calculation), points out the undecidability of the extension problem (using Hibert's 10^{th} problem), and deas with issues of diagrams of spaces and some situations involving finite group actions.

Together these results change one's point of view of an important area of mathematics. The result of polynomial time calculation of Postnikov systems indicates that the relevant function spaces have much lower complexity than one would predict on the basis of entropy and reated considerations. I consider this a result of the first order.

Reviewer's questions for the habilitation thesis defence (number of questions up to the reviewer)

Have you considered the possibility of using computational complexity results of the sorts you've proved to shed light on the analytic sizes of solutions to problems as measured by sizes of derivatives? Can this be used to improve on numerical algorithms in e.g. nonlinear PDE?

Conclusion

The habilitation thesis entitled Algorithmic aspects of topological problems

by Algorithmic aspects of topological problems ***fulfils*** requirements expected of a habilitation thesis in the field of algorithmic geometry/topology.

In Brno on

May 6, 2018