

## Příloha 7: Posudek oponenta habilitační práce

**Masarykova univerzita**

**Fakulta**

Přírodovědecká

**Habilitační obor**

Matematika – matematická analýza

**Uchazeč**

RNDr. Michal Veselý, Ph.D.

**Pracoviště**

ÚMS PřF MU

**Habilitační práce**

Solution spaces of almost periodic homogeneous linear difference and differential systems

**Oponent**

prof. RNDr. Jaroslav Jaroš, CSc.

**Pracoviště**

Univerzita Komenského v Bratislavě, FMFI

### Text posudku (rozsah dle zvážení oponenta)

In the thesis, the concepts of almost periodicity and limit periodicity considered in the framework of pseudometric spaces are used to study a variety of problems for linear difference (resp. differential) systems. The motivation for this study comes from the observation that many oscillation phenomena in the nature are better described by mathematical models which have the form of difference or differential equations with almost periodic coefficients rather than the models with pure periodic data. In the recent years there has been a rapid development of the theory of such equations as is witnessed by a long list of references at the end of the thesis. Thus, the work of Dr. Veselý is surely in the present trend and the obtained results are not only of interest to mathematicians working in the area of discrete or continuous dynamical systems, but also to scientists from other fields such as biology, musicology and others.

The definition of almost periodicity used here is a modified version of the definition by Harald Bohr (which is shown to be equivalent with S. Bochner's one) according to which a function  $f(x)$  on  $\mathbb{R}$  is almost periodic if it is continuous and for every  $\varepsilon > 0$  the set of all  $\varepsilon$ -translations is relatively dense, which means that there is some length  $c$  such that every interval of length  $c$  contains an  $\varepsilon$ -translate.

The purposes and main goals of the thesis are well described in the Abstracts of Chapters 1-7 in the introduction and will not be repeated here.

Among typical results obtained by Dr. Veselý and his most significant contributions to the theory of almost periodic linear difference and differential systems I choose to mention:

- developing a method for construction of almost periodic and limit periodic sequences having specific properties and its application to almost periodic difference systems;
- an analysis of almost periodic homogeneous linear difference systems which have no non-trivial almost periodic solutions;
- an introduction of the concepts of the so-called transformable and weakly transformable groups of matrices and their use in establishing conditions ensuring that in any neighbourhood of each considered system there exists a system which does not

possess a nontrivial almost periodic solution;

- the proof of the result which states that for any countable and totally bounded set there exists a limit periodic sequence whose range is exactly this set;
- construction of almost periodic functions with prescribed properties in a pseudometric space and its application in proving that in any neighbourhood of a skew-Hermitian (resp. skew-symmetric) differential system, there exists a system which does not possess an almost periodic solution other than the trivial one.

From a formal point of view, the thesis is written clearly, without errors and in a perfect English. It is very well organized, the results are new and interesting and their proofs are clear and easy-to-follow (though I must admit that some of them are very technical). The work shows that Dr. Michal Veselý is able to perform highly qualified mathematical research and to present the results of his investigation in a cultivated way.

I evaluate highly the fact that the thesis is not just a collection of reprints of the papers by Dr. Veselý, but it is a true monograph on the subject. It is a welcomed addition to the literature on almost periodic sequences and functions in pseudometric spaces and a valuable reference for anyone who is interested in properties of such sequences and functions.

#### **Dotazy oponenta k obhajobě habilitační práce (počet dotazů dle zvážení oponenta)**

1. In Chapters 5-7 concerning linear differential systems I have not found a continuous analogue of the notion of an asymptotic almost periodicity studied in the discrete case in subsection 4.3. Could you give a precise definition of an asymptotically almost periodic function and indicate what kind of results can be expected for linear differential systems with coefficients belonging to such a class of functions?
2. It is known that the notion of almost periodic functions in the sense of Bohr can be generalized to the so-called Besicovitch almost periodicity. (Roughly speaking, the space of Besicovitch almost periodic functions consists of the functions on  $\mathbb{R}$  which can be approximated „arbitrarily well“ by trigonometric polynomials under the Besicovitch seminorm.) Is it possible to extend the theory developed in this thesis (or at least some parts of it) also to this more general framework?

#### **Závěr**

Habilitační práce dr. M. Veselého „Solution spaces of almost periodic homogeneous linear difference and differential systems“ *splňuje* požadavky standardně kladené na habilitační práce v oboru matematická analýza.

V Bratislave, dne 08.04.2016

Prof.RNDr. Jaroslav Jaroš, CSc.